

Quadrupole deformation in Λ -hypernuclei

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Abstract. Shapes of light normal nuclei and Λ -hypernuclei are investigated using relativistic mean field approach. The FSUGold parametrization is used for this purpose. The addition of a Λ is found to change the shape of the energy surface towards prolate. The deformation in a Λ -hypernucleus, when the hyperon is in the first excited state, is also discussed. The effect of the inclusion of the hyperon on the nuclear radius is generally small with one exception.

PACS. 21.80.+a Hypernuclei

1 Introduction

One of the unique and interesting aspects of hypernuclei is the structural change caused by the hyperon. As an impurity in normal nuclei, a hyperon may be expected to induce many effects on the core nucleus, such as change in size[1], shape change, modification of its cluster structure[2], occurrence of nucleon and hyperon skin or halo[2,3], shift of neutron drip line to more neutron-rich side[3,4,5] etc. Owing to recent experimental developments some of those have been observed in light p-shell hypernuclei. As examples, we can refer to the reduction of B(E2) in ${}^7_\Lambda\text{Li}$ [6] and the identification of the super-symmetric hypernuclear state in ${}^9_\Lambda\text{Be}$ [7]. We can expect that a new experimental facility of Japan Proton Accelerator Research Complex (J-PARC) will reveal new spectral information on p and sd shell as well as neutron-rich hypernuclei.

As the shape of nuclei plays a decisive role in determining their properties, such as quadrupole moments and radii, mean field calculations have been performed in recent years to investigate the change of nuclear shape due to the addition of a Λ hyperon. Deformed Skyrme- Hartree-Fock(SkHF) studies in Ref. [8] have shown that the deformation of any hypernucleus is slightly less than the corresponding core nucleus. On the other hand, relativistic mean field (RMF) study in Ref. [9] found that the deformation completely disappears in ${}^{13}_\Lambda\text{C}$ and ${}^{29}_\Lambda\text{Si}$ hypernuclei though the corresponding normal core nuclei are deformed. Recently, a study with anti-symmetrized molecular dynamics by Isaka *et al.*[10] have found that the Λ in p-wave enhances the nuclear deformation, while that in s-wave reduces it.

To perform a systematic and quantitative study of the structure change of the p and sd-shell nuclei, caused by the hyperon, we study the binding energy, quadrupole deformation, and the root mean square radii of a number of hypernuclei within a RMF model. The relativistic mean field theory of nucleus has been fairly successful in reproducing the properties of finite nuclear systems[11]. It has also been extensively applied to the study of hypernuclei[12].

2 Results

There have been a number of RMF parametrizations for prediction of the nuclear ground state properties. In the present work the FSUGold Lagrangian density has been employed[13]. This parametrization has already been extended to include hyperons and to study the properties of hypernuclear systems[14]. In this work it is used to study the change in quadrupole deformation due to addition of hyperon.

The parameters of the ΛN interaction have been determined by fitting the experimental separation energies of 12 hypernuclei in the mass region 16 to 208[14]. The masses of the two physical mesons have been taken from experiment. The nucleon mass is taken as 939 MeV. The mass of the Λ has been fixed at 1115.6 MeV.

In the present work, we have chosen to limit the type of deformation to azimuthally symmetric and reflection symmetric systems which corresponds to prolate and oblate ellipsoids for quadrupole deformation. This eliminates all the three-vector components of the boson fields. Therefore, the limitation on the type of deformation simplifies the calculation with contributions from only the scalar meson, the zero components of the isoscalar vector meson, the photon, and the neutral ρ meson fields. This is the same set of boson fields that was required for spherical

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nuclei; however, these boson fields now have an additional angular dependence.

Solving the field equations with no further simplification can be very involved due to the difficulties encountered in obtaining solutions for coupled partial differential equations. We expand the boson fields in terms of Legendre polynomials and The nucleonic orbital wave functions in terms of spherical angle functions. The method of solution has been explained in detail in Ref. [15]. Terms up to angular momentum $L = 6$ have been taken into account. As we concentrate on very light nuclei, pairing is not expected to play a very important role. The grid size for solving the differential equations is taken to be 0.1 fm.

We calculate the quadrupole deformation parameter and the root mean square radii for a number of Λ -hypernuclei up to $A < 30$ putting the Λ in its deformed ground state and also in the first excited state, which have opposite parities. The results of our calculation are presented in Table 1.

In our calculation, the ground state of Λ is a pure s -state as the higher positive parity states are too high in the continuum. The first excited state is a negative parity state which has contributions from both the p orbitals. We thus indicate the solutions for hypernuclei with the hyperon in the ground or the first excited states with ‘s’ and ‘p’ in parentheses in the table and in the relevant figures. We have looked for both prolate as well as oblate minima in normal nuclei and hypernuclei. It is observed that in all the cases, the deformed hypernuclear minimum corresponds only to prolate deformation. Thus, in case of the excited state, the hyperon occupies the $K^\pi = 1/2^-$ state. No oblate minima are observed in the hypernuclear systems. This observation will be elaborated later in this work. Very little information is available for the Λ separation energy in the ground state of the hypernucleus; the existing experimental values agree reasonably well with our calculation.

We see that when the Λ is placed in the ground state, the deformation changes slightly on inclusion of the hyperon, with only one exception. The deformation may increase as well as decrease, but in general the shape remains similar to that of the core nucleus.

However, in the case of $^{13}_\Lambda\text{C}$, the shape appears to change drastically on the inclusion of the Λ . The ground state of ^{12}C shows oblate deformation, whereas the corresponding hypernucleus $^{13}_\Lambda\text{C}$ is prolate.

This necessitated further investigation where we study the total energy of the system against deformation in a constrained calculation. In Fig. 1 we have represented the total binding energy with respect to the minimum ($E_{tot} - E_{tot}^{min}$) for ^{12}C and $^{13}_\Lambda\text{C}$ as a function of deformation parameter β . We see that the energy surface of ^{12}C is almost flat with a oblate minimum around $\beta = -0.2$. Though there is no minimum for prolate deformation, around $\beta = 0.1$ there exists a region which might have formed a minimum. Inclusion of the Λ makes the oblate minimum disappear and the prolate minimum is found to be formed in the energy surface of $^{13}_\Lambda\text{C}$. One can see that although the unconstrained calculations predict the ground state of ^{12}C

Table 1. Calculated Binding energy/nucleon ($-E/A$) and Λ separation energy (E_Λ) in MeV, quadrupole deformation parameter β , and the root mean square radius in fm at the minimum energy for the core nucleus and the corresponding hypernucleus in different Λ states.

Nucleus (Λ -state)	$-E/A$		β	r_p	r_n	$\sqrt{\langle r^2 \rangle}$
^8Be	5.668	-	0.38	2.41	2.31	2.36
	5.533		-0.37	2.39	2.28	2.33
$^9_\Lambda\text{Be}$ (s)	5.837	7.189*	0.34	2.40	2.56	2.34
^{10}B	6.201	-	0.27	2.41	2.26	2.33
	6.116		-0.20	2.39	2.24	2.32
$^{11}_\Lambda\text{B}$ (s)	6.501	9.501	0.33	2.54	2.49	2.40
^{12}C	7.175	-	-0.21	2.36	2.18	2.27
$^{13}_\Lambda\text{C}$ (s)	7.534	11.842†	0.13	2.59	2.39	2.39
$^{13}_\Lambda\text{C}$ (p)	6.683	0.779	0.14	2.61	3.41	2.92
^{18}F	7.562	-	0.14	2.70	2.45	2.58
	7.561		-0.11	2.69	2.44	2.57
$^{19}_\Lambda\text{F}$ (s)	7.883	13.661	0.13	2.71	2.55	2.56
$^{19}_\Lambda\text{F}$ (p)	7.399	4.465	0.14	2.71	2.69	2.63
^{22}Na	7.565	-	0.22	2.85	2.57	2.72
	7.475		-0.13	2.83	2.55	2.69
$^{23}_\Lambda\text{Na}$ (s)	7.904	15.362	0.21	2.86	2.65	2.70
$^{23}_\Lambda\text{Na}$ (p)	7.537	6.921	0.22	2.86	2.74	2.74
^{24}Mg	7.814	-	0.25	2.91	2.60	2.76
	7.640		-0.15	2.89	2.59	2.74
$^{25}_\Lambda\text{Mg}$ (s)	8.142	16.014	0.23	2.91	2.67	2.74
$^{25}_\Lambda\text{Mg}$ (p)	7.823	8.039	0.24	2.91	2.75	2.78
^{25}Mg	7.872	-	0.20	2.88	2.64	2.76
	7.793		-0.13	2.87	2.63	2.75
$^{26}_\Lambda\text{Mg}$ (s)	8.248	17.648	0.22	2.92	2.70	2.75
$^{26}_\Lambda\text{Mg}$ (p)	7.927	9.30	0.23	2.92	2.78	2.79
^{25}Al	7.657	-	0.15	2.95	2.56	2.77
	7.565		-0.11	2.94	2.55	2.76
$^{26}_\Lambda\text{Al}$ (s)	8.001	16.601	0.14	2.95	2.63	2.75
$^{26}_\Lambda\text{Al}$ (p)	7.676	12.103	0.15	2.95	2.71	2.78
^{26}Al	7.828	-	0.11	2.92	2.60	2.76
	7.795		-0.09	2.92	2.59	2.76
$^{27}_\Lambda\text{Al}$ (s)	8.200	17.872	0.13	2.96	2.66	2.76
$^{27}_\Lambda\text{Al}$ (p)	7.874	9.07	0.14	2.96	2.74	2.80
^{28}Si	8.080	-	0.00	2.95	2.59	2.77
$^{29}_\Lambda\text{Si}$ (s)	8.453	18.897	0.06	3.00	2.65	2.78
$^{29}_\Lambda\text{Si}$ (p)	8.118	9.182	0.06	3.00	2.72	2.86

*Exp. value is 6.84 ± 0.05 MeV[16]

†Exp. values are 11.69 ± 0.12 MeV[16], 11.38 MeV[7]

and $^{13}_\Lambda\text{C}$ to be oblate and prolate, respectively, actually a flat minimum is being replaced by a more sharp prolate one. Our result is different from the previous HF calculations by Zhou *et al.*[8] or the SkHF+BCS calculations by Win *et al.*[17], which have reported similar deformations for ^{12}C and $^{13}_\Lambda\text{C}$. Some calculations have also reported a spherical configuration for $^{13}_\Lambda\text{C}$ [9,10,18].

As a case where deformation appears in hypernuclei while the core is spherical, we study $^{29}_\Lambda\text{Si}$. Lu *et al.*[18] concluded that the prediction of the shape of $^{29}_\Lambda\text{Si}$ is parameter dependent. We see that $^{29}_\Lambda\text{Si}$ is slightly prolate deformed with the FSUGold parameter whereas ^{28}Si is

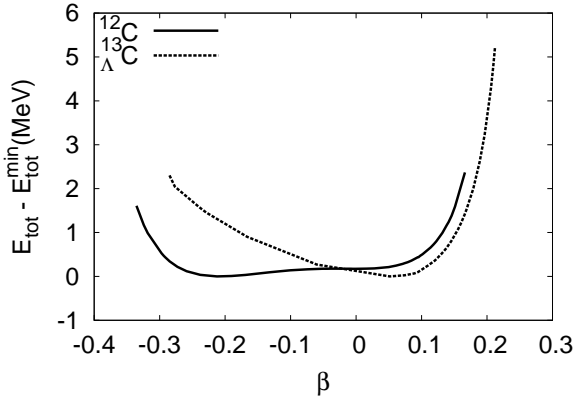


Fig. 1. Total energy with respect to the minimum ($E_{tot} - E_{tot}^{min}$) as function of deformation β for ^{12}C and the corresponding hypernucleus $^{13}_{\Lambda}\text{C}$.

spherical. In Fig. 2 we have represented the total binding energy with respect to the minimum ($E_{tot} - E_{tot}^{min}$) for ^{28}Si and $^{29}_{\Lambda}\text{Si}$ as a function of deformation. From Fig. 2,

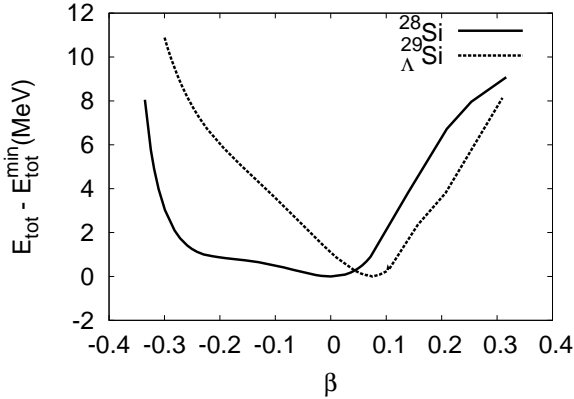


Fig. 2. Total energy with respect to the minimum as function of deformation β for ^{28}Si and the corresponding hypernucleus $^{29}_{\Lambda}\text{Si}$.

we see that the rather flat energy minima at the spherical configuration in case of ^{28}Si , becomes sharper and shifts slightly towards positive side in case of $^{29}_{\Lambda}\text{Si}$.

We also note that our results for $^{25,27}_{\Lambda}\text{Mg}$ agree with the SkHF+BCS calculations by Win *et al.*[17]. However, our results, as a whole, are different from the previous RMF and SkHF calculations[9,17,4,8,18,3] in the sense that all the previous calculations report a decrease in deformation on addition of a Λ particle. Our calculations show that the deformation may increase in some cases. In general, the inclusion of the Λ hyperon makes the energy surface sharper, the oblate minima disappears and the prolate one becomes prominent, thus producing a shift in deformation. Similar observations have been found for the other nuclei investigated in Table 1.

In the case of excited Λ states, our calculations show that when the Λ goes from the ground-state to the p-state

it slightly increases the deformation excluding the case of $^{29}_{\Lambda}\text{Si}$, where the deformation remains the same. Calculation for the deformation of hypernuclei for an excited Λ state reported by Isaka *et al.*[10] found similar results. In $^9_{\Lambda}\text{Be}$ and $^{11}_{\Lambda}\text{B}$, the excited Λ state is unbound.

We have also investigated the changes in the root mean square radii of the hypernuclei (Table-1). The last column, in case of hypernuclei, includes the effect of the hyperon also. It is seen that the inclusion of a Λ hyperon, causes either a slight increase or no modification in neutron and proton radii (r_n and r_p , respectively) when the Λ particle is in the s-state. When the hyperon is excited to the p-state the size of the hypernucleus increases only slightly in almost all cases. The only exception is $^{13}_{\Lambda}\text{C}$, where this increase in radius is significant, so that the hypernucleus is much larger than the corresponding core nucleus.

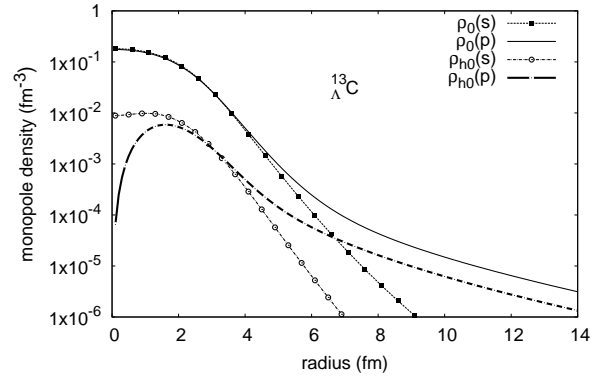


Fig. 3. The monopole nucleon and hyperon density profile in $^{13}_{\Lambda}\text{C}$ when the Λ is in different single particle state (s and p, given in parentheses). Here ρ_o refer to monopole nucleon density and ρ_{h0} refer to monopole hyperon density. .

The reason for the large increase in size in $^{13}_{\Lambda}\text{C}$ can be seen in the distribution of densities of the nuclei. We plot the monopole densities in $^{13}_{\Lambda}\text{C}$ and $^{25}_{\Lambda}\text{Mg}$ in Figs. 3 and 4, respectively, to see the effect of the hyperon. Both the cases with the hyperon in its ground state and excited state are shown. Fig. 3 shows that in $^{13}_{\Lambda}\text{C}$, when the hyperon is placed in the p-state, the densities extend to larger distances. Actually, the hyperon is very loosely bound in the excited state. Hence, its wave function extends to a very large distance. As the hyperon interacts strongly with the nucleons, the nucleon density also shows a large tail, reminiscent of halo nuclei.

However, this is not a general phenomena as evident from Table I. As an example, we present the density profile for $^{25}_{\Lambda}\text{Mg}$ in Fig. 4. It is clear that in this hypernucleus, both the nucleon and the hyperon densities remain practically unchanged whether the hyperon is in the s-state or in the p-state. The p-shell hyperon in this case is strongly bound and does not show any halo-like behaviour.

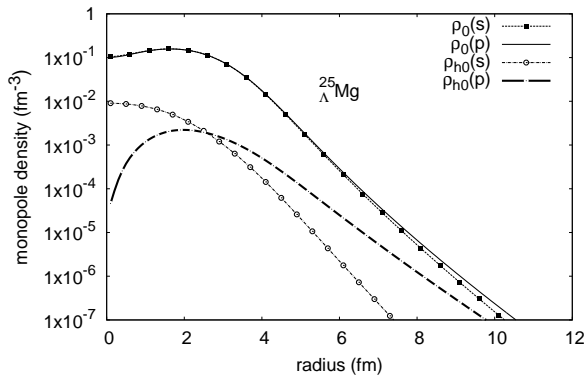


Fig. 4. The monopole nucleon and hyperon density profile in $^{25}_{\Lambda}\text{Mg}$ when the Λ is in different single particle state (s and p). See caption of Fig. 3 for details.

3 Summary

We have studied the deformation of core and hypernuclei in the RMF approach using the FSUGold parameter set. The calculated Λ binding energies agree reasonably well with the experimentally observed values. We see that the inclusion of a Λ hyperon changes the energy surface making it steeper. There exists no oblate minimum in any light hypernucleus. Results of the present calculation differ from the previous ones. The latter usually predict that inclusion of a Λ tends to drive the shape to spherical, while our results show that the change in β is usually small and may go either way. In general, the nucleon density profile changes to a small extent on inclusion of the hyperon, whether in the ground state or the first excited state. When the Λ goes to the p-state, both the deformation and radius increases by a small amount. The only exception is $^{13}_{\Lambda}\text{C}$ where, the hyperon being very loosely bound, create a halo-like structure.

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